AIAA Journal
Vol. 35, No. 3, March 1997

Optimal Placement of Piezoelectric Actuators for Active Noise Control

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Optimal placement and size of disk-shaped piezoelectric actuators is studied to reduce the radiation of sound into the space above a plate structure when excited by an acoustic pressure field produced by a noise source located below the plate. Finite element modeling is used to simulate the piezoelectric active structure. The objective of the optimization procedure is to minimize the sound energy radiated onto a hemispherical surface of given radius, and the design parameters are the locations and sizes of the piezoelectric actuators, as well as the amplitudes of the voltages applied to them. To allow for variations in the locations of piezoelectric actuators on a flat plate, an automatic mesh and boundary condition generation program is developed based on the idea of moving mesh templates for complicated regions near piezoelectric actuators. This new contribution enables the optimal design of active plate structures for noise control problems. Numerical results of the optimal design at the resonance and off-resonance frequencies show remarkable noise reduction, and the optimal locations of the actuators are found to be closer to the edges of the plate. The optimized results are robust such that when the acoustic pressure loading is changed, the radiated sound is still reduced. The robustness of the design is studied by fixing the position of the actuators and optimally adjusting the actuator voltages to obtain broadband noise reduction.

Nomenclature

BSIDE	= size parameter of the mesh template
$\boldsymbol{b}$	= design variable vector
b_i	= ith design variable
c	= wave speed of the acoustic medium
$\boldsymbol{F}$	= point force on the structure
g_i	= ith inequality constraint
IX1CH, IX2CH, IY1CH, IY2CH	= attribute arrays of the discretized points in the automatic mesh generation
$\boldsymbol{K}_{uu}$	= stiffness matrix of the structure and the piezoelectric actuators
$\boldsymbol{K}_{u\varphi}$	= piezoelectric coupling matrix
$\boldsymbol{K}_{\varphi\varphi}$	= dielectric stiffness matrix
k	= wave number in the acoustic medium
L_i	= upper bound of ith design variable
$\boldsymbol{M}$	= mass matrix of the structure and the piezoelectric actuators
NOX	= number of points on the x axis for the mesh generation
NOY	= number of points on the y axis for the mesh generation
$p(r)$	= acoustic pressure at the observation point r on the hemisphere
R	= $\lvert r - r_s \rvert$
r	= location of the observation point on the hemisphere
r_s	= location vector on the surface S
$r1$	= radius of first piezoelectric element
$r2$	= radius of second piezoelectric element
S	= top surface of the structure
$\boldsymbol{s}$	= state variable vector
$t1$	= thickness of first piezoelectric element
$\boldsymbol{U}$	= nodal displacement of the structure and the piezoelectric actuators
U_i	= lower bound of ith design variable
$u(r_s)$	= normal displacement at point r_s on the surface S
W_{tot}	= total radiated sound power
X1PT, X2PT, Y1PT, Y2PT	= arrays that represent the discretized points on x and y axis for the mesh generation
X3PT	= discretized points at the interface boundary for the mesh generation
$x01$	= x coordinate of first piezoelectric element
$x02$	= x coordinate of second piezoelectric element
$y01$	= y coordinate of first piezoelectric element
$y02$	= y coordinate of second piezoelectric element
θ, ϕ	= angles of the observation point r on the hemisphere
ρ	= density of the acoustic medium
Φ	= nodal values of the electrical potential
Ψ	= objective function in optimal design
$\psi1$	= amplitude of applied voltage on the first piezoelectric element
$\psi2$	= amplitude of applied voltage on the second piezoelectric element
ω	= excitation frequency of the noise

Introduction

MUCH research has been done on methods for combating noise pollution caused by structures such as automobiles, aircraft, and buildings. In controlling noise, there are two distinct approaches: passive and active methods. Passive approaches are based on the design of material properties or shapes of the structure so as to minimize radiated noise. Exploiting damping layers in the structure is a good example in passive noise control.[1] Recent advances in computing resources and optimization procedures have made numerical optimization feasible for complex structures using active techniques.[2,3]

Smart materials or structures have emerged as promising active techniques to reduce the radiated sound field. In such structures, piezoelectric ceramics are widely used as active devices on the structures. Much research has been done analytically and experimentally for piezoelectric active, adaptive, or intelligent structures.[4–8] However, the design of smart structures for minimal sound radiation is a

Received Jan. 22, 1996; revision received Oct. 7, 1996; accepted for publication Oct. 15, 1996; also published in *AIAA Journal on Disc*, Volume 2, Number 2.

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multidisciplinary and challenging problem, which involves a complex model of the structure with active devices made of piezoelectric materials, understanding of structural acoustics, and large numbers of parameters that affect the active system performance.

Many factors affect the performance of the smart system, for example, the number, sizes, locations, and voltages applied to the actuators. Thus, efforts to optimize these parameters are essential to increase the performance of the system. One of these efforts is optimal placement of the actuators, which has been studied for the last two decades.[9–14] Some papers have dealt with the optimal location of distributed actuators.[12–14] Clark and Fuller[13] studied the location of a piezoelectric actuator and both the size and location of a polyvinylidenefluoride (PVDF) sensor for active structural acoustic control. Wang et al.[14] have presented a formulation of the optimization problem for the placement and sizing of piezoelectric actuators in adaptive least mean square (LMS) control systems. Besides the optimal location, the optimal size of the piezoelectric actuators and sensors must be investigated to achieve genuine performance improvement. However, it is not so easy to find the optimal configuration of the piezoelectric active structures. For radiated structural noise control, piezoelectric active structure could have different optimal configurations at different frequencies, for example, resonance and off-resonance frequencies. This means that the size and the location of the actuator should be carefully configured because the configuration cannot be changed if the excitation frequency is changed. Only the voltage applied to the actuator can be adjusted in practice. Thus optimal placement of the actuator should result in broadband performance with respect to radiated noise control.

According to the previous study,[15,16] with an optimal configuration for a single piezoelectric actuator, a remarkable reduction of the radiated noise was observed for a moderate frequency band. The sound radiation of the structure at the first few natural frequencies can be controlled by changing the voltage applied to a single piezoelectric actuator. To control radiation at higher frequencies, the use of multiple piezoelectric actuators is suggested. Therefore, this paper deals with optimal design of the multiple piezoelectric actuators with plate structure for reducing radiated noise for a higher range of frequencies.

Rectangular piezoelectric actuators that are easy to model are used most often. But it is known that circular disk-shaped actuators produce a higher bending moment in the radial direction, which has advantages in controlling the vibration of plate structures. In modeling plate structure with piezoelectric devices, many approaches have been proposed: pure bending deformation model,[17] symmetrical piezoelectric thick shells model in finite element analysis,[18] a hybrid equivalent single layer theory[19] or layerwise theory,[20] and the finite element method by using three-dimensional piezoelectric, transition, and shell elements.[21] Within these modeling approaches, it is advantageous to use the finite element method because this method can be used for any structures and anisotropic materials, like piezoelectric media, with very good accuracy. Therefore, in this paper, the finite element method[21] is used.

When several piezoelectric actuators are considered and we wish to change their location on the plate, a special automatic mesh generation is necessary. When the locations of the devices are moved during the optimization process, the finite element mesh has to be automatically generated. Commercially available mesh generation packages are not suitable for this problem because electric boundary conditions on the piezoelectric devices are complicated and it is difficult to generate these conditions during the optimization process using general mesh generation programs. The mesh generation program has to be closely linked with the analysis program and the input parameters. For example, dimensions of the plate and locations and size of piezoelectric devices have to be conveyed to the mesh generation program, which need special program links and consume much computation time. To cope with these features, a new program for automatic mesh generation has been developed that is customized for arbitrary locations of piezoelectric devices on flat plate structures.

To date, in most active structural acoustic control studies point loading of the structure is considered and this is not realistic. Noise distribution on the structure is hard to determine in a practical situation because there is interaction between the structure and the surrounding acoustic medium. To find the actual structural response, the structure, the acoustic medium that surrounds the noise source, and the structure with appropriate boundary conditions should be included in the analysis. This results in a complicated problem. The actual acoustic pressure distribution on a structure may not be known in many practical applications and it may also be highly variable. But if the design of a smart structure is robust for many different patterns of acoustic loading, the performance may be acceptable in practice. Therefore, Kim et al.[15] and Varadan et al.[16] have considered acoustic pressure loading in a comparative manner: a loading pattern is used, and after optimal design is performed, different patterns of acoustic loading, all of which have the same total input power, are examined to see the robustness of the resultant optimal design. In this paper, the same method is adopted but for multiple actuators.

This paper aims at the reduction of radiated noise from piezoelectric active structures in higher bands of frequencies by optimally designing the location of piezoelectric actuators. Two piezoelectric actuators are considered, and not only the location and the applied actuator voltage but also the thickness and radius of the actuators are optimally designed. Figure 1 shows the geometry of the problem. A flat plate structure is built into an infinite baffle, and the plate is excited by a noise source below the plate. The finite element method is used to model the actuators and the plate structure taking into account all the relevant dynamical fields in the piezoelectric actuators and the structure. This method uses a combination of three-dimensional piezoelectric, transition, and shell elements. The distribution of the acoustic loading on the plate is assumed to be a constant or of a given functional form. This distribution is converted into an equivalent nodal force distribution on the structure. The structural response is computed with the acoustic loading on the plate structure and an rf voltage applied to the piezoelectric actuator

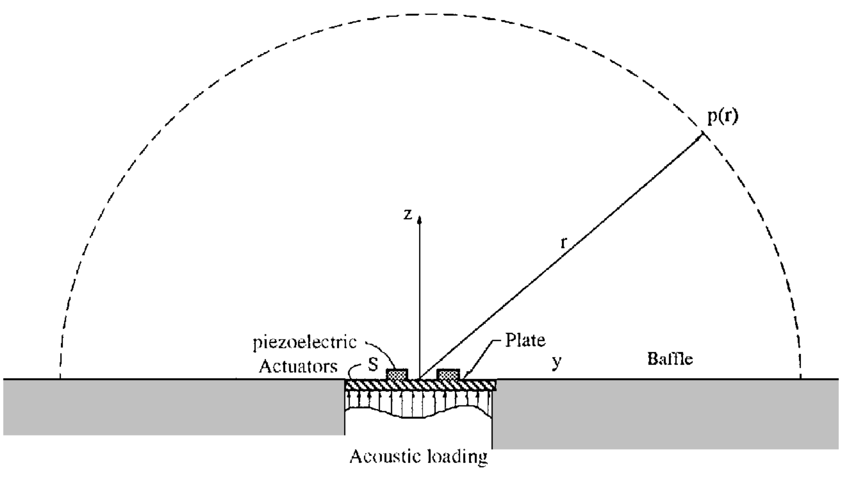


Fig. 1 Plate structure with PZT actuator in an infinite baffle.

by using the finite element method. At this stage, the effect of an infinite acoustic medium above the plate is assumed to be negligible, and traction-free boundary conditions are applied to the top surface of the structure. The acoustic field radiated into the exterior region is then computed using a Helmholtz integral representation. For the calculation of the integral representation, the surface nodal values of the displacement resulting from the finite element method are used. An optimization technique is used to minimize the radiated sound by rearranging the sizes and the locations, as well as the amplitudes, of the voltages applied to the actuators. The minimizing cost function is the integrated value of the radiated sound energy on a hemisphere of desired radius whose base includes the structure. After finding an optimal design configuration, several kinds of acoustic loading distributions are examined to check the robustness of the optimal result. The frequency stability of the optimal design is studied by optimizing at a resonance frequency and studying the noise reduction characteristics for a range of frequencies changing only the voltage applied to the actuators.

Modeling and Theory

A rectangular plate with piezoelectric actuators is built into an infinite baffle and an acoustic pressure disturbance is applied to the bottom of the plate (Fig. 1). Two separate stages of analysis of the problem are considered. First, the structural response generated by the acoustical pressure load using the finite element analysis is computed neglecting the influence of the acoustic medium above the plate. Next the radiated sound pressure and the total radiated sound power are found using the surface displacements of the structure resulting from the finite element analysis as the source terms in the integrand of a Helmholtz integral representation.

In the finite element model for the structure, three-dimensional piezoelectric elements are used in the piezoelectric regions and their neighbors and flat shell elements are used for the remaining part of the plate structure and transition elements connect the shell elements and three dimensional solid elements (see Fig. 2).

Formulation for the Piezoelectric Actuators and Structure

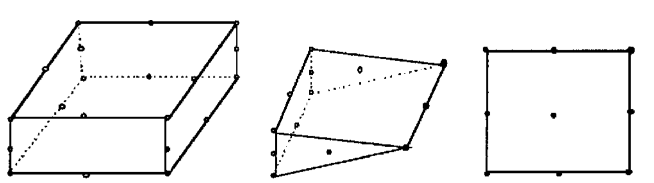
To solve the response of the structure, the finite element method is adopted because this method can be used for arbitrary geometry and includes the anisotropic properties of the piezoelectric material. Finite element equations of a piezoelectric actuator embedded on a structure has already been formulated[22,23] and can be written as

$$\left(-\omega^2\begin{bmatrix} \boldsymbol{M} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_{uu} & \boldsymbol{K}_{u\phi} \\ \boldsymbol{K}_{u\phi}^T & \boldsymbol{K}_{\phi\phi}\end{bmatrix}\right)\begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{\Phi}\end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F} \\ \boldsymbol{Q}\end{Bmatrix} \tag{1}$$

There is no distinction between the piezoelectric medium and structural medium in applying Eq. (1) except that the piezoelectric coupling matrix and the dielectric stiffness matrix are zero in the structure. The stiffness and mass matrices of flat shell and transition elements for the structure are already described in the previous work.[21,30] The present work referred to finite element formulations in Refs. 24–26. The acoustic pressure distribution applied to the bottom of the plate is converted into the point forces and the electrical potential is applied across the electrodes of the piezoelectric actuators.

Acoustic Pressure Loading

The actual distribution of acoustic pressure disturbance on the structure is hard to determine. Therefore, different patterns of acoustic loads that have the same total input power have been investigated: constant, random, Bessel function of zero order, Bessel function of first order, and the sound field distribution produced in an acoustically hard box. The pressure distribution in an acoustically hard box is expressed in terms of a modal superposition and evaluated on the top surface of the box of dimensions $305 \times 305 \times 305$ mm when an acoustic point source is located at the bottom center of the box. In the optimal design procedure, constant acoustic pressure loading is assumed, and after finding an optimal configuration, other loading conditions are examined to show the robustness of the design result for suppressing or reducing radiated noise to the exterior of the box.



20-node three-dimensional element **13-node transition element** **9-node shell element**

Fig. 2 Three-dimensional, transition, and shell finite elements.

Radiated Sound Pressure and Total Radiated Sound Power

Finite element analysis is used to compute the normal displacements, $\boldsymbol{n} \cdot \boldsymbol{u}$, on the top surface of the structure S, which can then be used to compute the cost function in terms of the radiated sound energy. We assume that the plate is built into an infinite baffle. This assumption may not be true in real problems. However, its results can give some physical insight in controlling radiated sound from real enclosures. Using the normal displacement on the surface S of a vibrating structure, the radiated pressure can be expressed as

$$p(r) = -\frac{\rho\omega^2}{4\pi}\int_s u(r_s)\frac{e^{ikR}}{R}\,\mathrm{d}s \tag{2}$$

Equation (2) is solved numerically by a commonly used Gauss numerical integration scheme.

The goal of optimal design in this paper is to minimize the sound radiated from the structure. There could be several ways to describe this goal. In the ideal case, the radiated sound pressure should be minimized at every point. However, this is not so easy. Therefore, a more global approach is used. The total radiated sound power on a hemispherical surface of radius r is defined as the cost function that is to be minimized. It is given by

$$W_{\text{tot}} = \int_0^{2\pi}\int_0^{\pi/2}\frac{|p(r)|^2}{2\rho c}r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi \tag{3}$$

This integral is computed by using an extended Simpson numerical integration rule[27] for the dual integration on the θ and ϕ variables on a surface of constant radius r.

Automatic Mesh Generation

For optimal design of smart structures, the locations, sizes, and gains of the piezoelectric actuators must be varied. One of the challenges in the design is the need for an automatic mesh generation program to allow for arbitrariness in the locations and sizes of the piezoelectric devices. By moving a mesh template that includes a piezoelectric device, we have developed an automatic mesh generation program. This is similar to the domain parameterization method that is introduced for the treatment of shape variation problems.[28] The plate structure is assumed to be of flat, rectangular shape, and two circular disk-shaped piezoelectric elements are considered to be mounted on one side of the plate. Most of the plate structure is meshed by using a nine-node plate element except for the piezoelectric devices and small neighboring regions, which are modeled by the element template. Figure 3 shows the layout of the plate and details of the element template. The template includes the most complicated part of the mesh with a combination of elements—20-node three-dimensional, 13-node transition, and 9-node plate elements. The size of template is determined by the length BSIDE, which is proportional to the radius of the piezoelectric patch. Two piezoelectric devices are allowed to move on the flat plate with the assumption that the templates never overlap, so that the template shape is not changed. Under these conditions, automatic node and element generation algorithms are developed. Details of the algorithms are shown in Appendix A.

Optimization

When the piezoelectric actuators are used for active noise control, there are many undetermined factors: number, locations, sizes of the actuators, and the excitation voltages. These can be taken as design variables in the optimization procedure. For simplicity, the number of piezoelectric devices is restricted to two. This results in a total of 10 design variables.

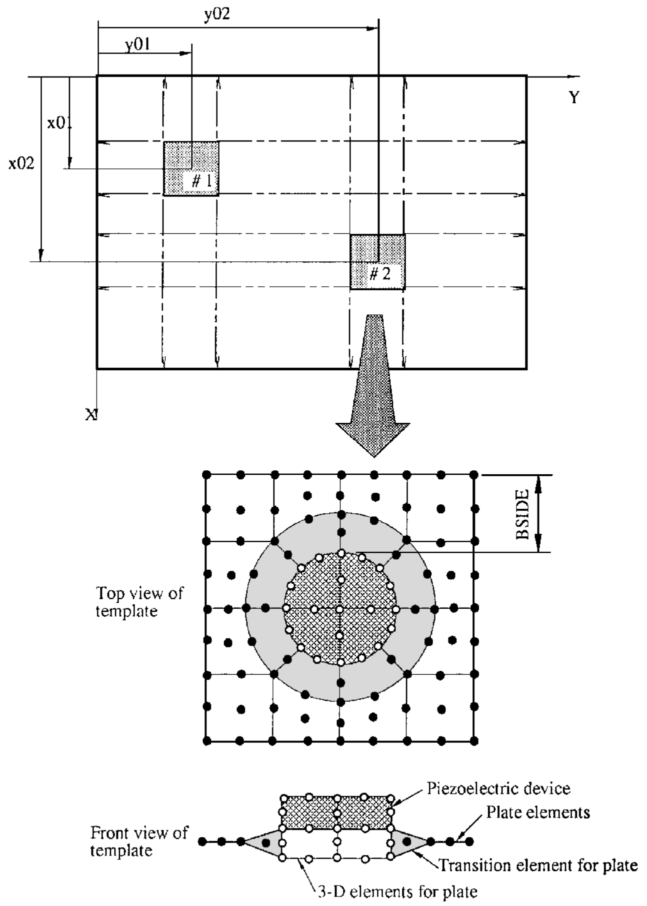


Fig. 3 Finite elements for plate structure with piezoelectric actuators.

The objective function in optimal design is the total radiated sound power,

$$\Psi = \min W_{\text{tot}}(\boldsymbol{b}, \boldsymbol{s}) \tag{4}$$

Design variables are automatically changed to achieve the goal and these variables should be restricted in some manner to be practical. For this, inequality conditions are used, such as

$$L_i \le b_i \le U_i \tag{5}$$

The preceding inequality conditions can be expressed as two inequality constraints

$$g_{(i-1)*2+1} = b_i - L_i, \qquad g_{i*2} = U_i - b_i \tag{6}$$

Another inequality constraint is that the templates for the piezoelectric devices do not overlap. Therefore, a total of 21 inequality constraints are involved.

Figure 4 shows the flowchart of the optimal design procedure. With the constant acoustic pressure loading and activated piezoelectric actuators, the response of the structure is found using the finite element method. From this response, the sound radiated from the structure and the total radiated sound power is calculated. Optimization is performed until the result has converged yielding the required value of the cost function.[29] A sequentially unconstrained minimization technique is used for the constraints, and Powell's method is applied to find the minimum point in each unconstrained minimum search. A feature of this program is the automatic generation of weighting factors for the inequality and equality constraints functions such that all constraints are normalized at the base point. This tends to reduce the possibility that one or more of the constraints will dominate the penalty function. Because Powell's direct search method in unconstrained minimization problems is incorporated in this program, it does not require the derivation or use of partial derivatives of objective function and constraints.

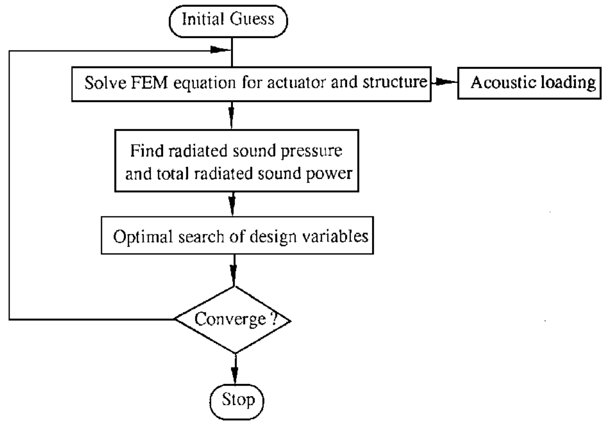


Fig. 4 Flowchart for the optimization algorithm

Numerical Examples

A square aluminum plate in an infinite baffle is considered (Fig. 1). An acoustic pressure load is applied at the bottom of the plate and piezoelectric actuators are bonded on top of the plate. The size of the aluminum plate is 305 × 305 mm and the thickness is 0.8 mm. The material properties of PZT-5 (lead zirconate titanate) are used for the piezoelectric actuators (material properties are listed on Appendix B). Clamped boundary conditions are used for the outside edges of the plate. To model the plate, shell and three-dimensional solid elements with transition elements that can connect shell and solid regions are adopted. The accuracy of this modeling approach has been shown in previous work.[15,16]

Automatic Mesh Generation

An automatic mesh generation program has been developed to allow for variations in the locations of the piezoelectric devices. Given the locations, radii, and thicknesses of piezoelectric devices and the size ratio of template vs radius of piezoelectric devices (BSIDE), nodal coordinates and element connectivity are generated automatically. This program is applicable to a flat plate structure with two circular piezoelectric devices on one side of the plate. For more general problems, such as multiple piezoelectric elements attached to both sides of the plate, curved structures, and arbitrary shapes of piezoelectric devices, some modifications of the program will be necessary.

Modal Characteristics of the Structure

Before performing the optimal design, the natural frequencies of the structure are determined to find the contribution of the different modes to the radiated noise. For example, when the noise frequency is matched with the natural frequency of the structure, the mode corresponding to the natural frequency has a dominant effect on the vibration and perhaps the radiated noise. At a resonance frequency, many successful attempts at noise reduction have been reported.[6,7,13–16] However, once the excitation frequency is far from the natural frequency, the vibration of the structure is not affected by a dominant natural mode but by a combination of many natural modes. This means that to control the noise radiation at an off-resonance frequency, related natural modes should be controlled, which is more difficult than the on-resonance case.

When the piezoelectric actuators are mounted on the plate, the natural frequencies will be changed because the stiffness as well as the mass matrices are different. Also the natural frequency will be altered when the location and size of the actuators are changed. Furthermore, when the piezoelectric actuators are activated, the stiffness matrix is also slightly changed. This means the piezoelectric active structures are no longer static structures that have fixed values of stiffness, mass, and damping. Therefore, when the active behavior of the piezoelectric actuators are considered, it is not so easy to find the natural frequency of the piezoelectric active structures. In contrast, this adjustment of the properties of the piezoelectric active structures can be a possible solution in the control of radiated noise.

Although the exact natural frequencies of the piezoelectric active structures are difficult to find, the theoretical natural frequencies of

Table 1 Natural frequencies of the aluminum plate in the absence of PZT actuators

Mode	Frequency
1, 1	74.06
2, 1	151.05
2, 2	222.71
1, 3	270.90
3, 1	272.01
3, 2	339.55
3, 3	452.74
4, 1	433.18
4, 2	499.37
4, 3	609.86

Table 2 Initial condition and optimized result at resonance, $f = 270$ Hz

Design variables	Initial condition	Intermediate result	Optimal result
$b1(x01)$	90 mm	81.05 mm	75.47 mm
$b2(y01)$	90 mm	79.81 mm	82.92 mm
$b3(x02)$	210 mm	219.64 mm	204.9 mm
$b4(y02)$	210 mm	218.37 mm	206.1 mm
$b5(r1)$	10 mm	8.87 mm	9.51 mm
$b6(r2)$	10 mm	9.44 mm	9.57 mm
$b7(t1)$	1 mm	1.01 mm	1.0 mm
$b8(t2)$	1 mm	0.96 mm	1.24 mm
$b9(\psi1)$	10 V	17.21 V	39.25 V
$b10(\psi2)$	10 V	17.84 V	4.94 V
Objective function-radiated sound power, dB	$0.105 \times E{-}6$ (30.74 dB)	$0.2783 \times E{-}7$ (24.96 dB)	$0.9108 \times E{-}9$ (10.1 dB)

the bare plate can be considered as reference values. Table 1 shows these values.[31] The size and material property of the plate structure are the same as before. The fourth and fifth natural frequencies are closely spaced (270 and 272 Hz) and correspond to (1, 3) and (3, 1) modes. For design at an on-resonance frequency, 270 Hz is chosen and 300 Hz is selected for the off-resonance design attempt.

Optimal Design at $f = 270$ Hz (On Resonance)

In the previous studies,[15,16] it was found that the optimal design with one piezoelectric device gives good results within a limited frequency range, up to 200 Hz. To cover higher frequencies, a design attempt with two piezoelectric actuators is proposed and the design is optimized at the fourth resonance frequency (270 Hz). Design variables are bounded as follows: 50 mm < b1, b2, b3, b4 (locations of piezoelectric devices) < 250 mm, 5 mm < b5, b6 (radius of piezoelectric devices) < 15 mm, 0.5 mm < b7, b8 (thickness of piezoelectric devices) < 1.5 mm, and −150 V < b9, b10 (amplitudes of applied voltages) < 150 V. The applied voltages on the piezoelectric actuators is out of phase with respect to the noise source, which means negative gain in displacement feedback.

As previously described, five different types of acoustic pressure load are considered. In the optimal design, a constant pressure load ($p = 2$ Pa) is assumed first, and after finding an optimal configuration, other loading conditions are examined with the optimal result. In applying different acoustic loading conditions, the input powers are confined to the same value. Table 2 shows the initial values of the design parameters and their optimized values. An intermediate result is shown in Table 2 during the optimization procedure. The value of the objective function is the total radiated power (watts) represented in decibel scale with a reference of 10^{-12} W.

Figure 5 represents the normal displacements at the top of the plate at the initial design parameters, and Figs. 6 and 7 show these displacements at the intermediate and optimal results. The vibration of the plate is reduced significantly by optimally designing the size of the PZT actuators and the voltages applied to them. Although the maximum normal displacements on top of the plate are not very different in Figs. 6 and 7, according to Table 2, the optimized result shows better reduction in total radiated power than the intermediate result. This can be explained by the fact that in the sound radiation of the flat plate structure, corners or edges contribute significantly to the

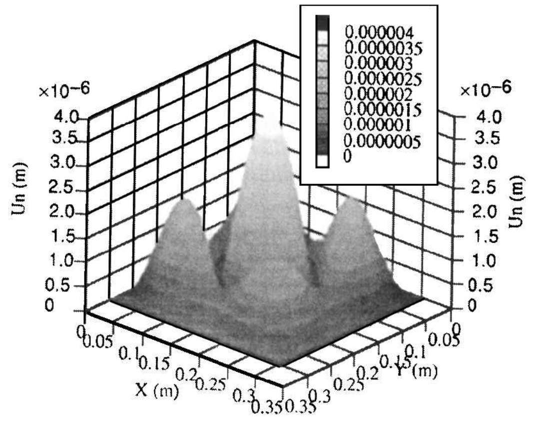


Fig. 5 Normal displacements at the top of the plate: initial condition, $f = 270$ Hz.

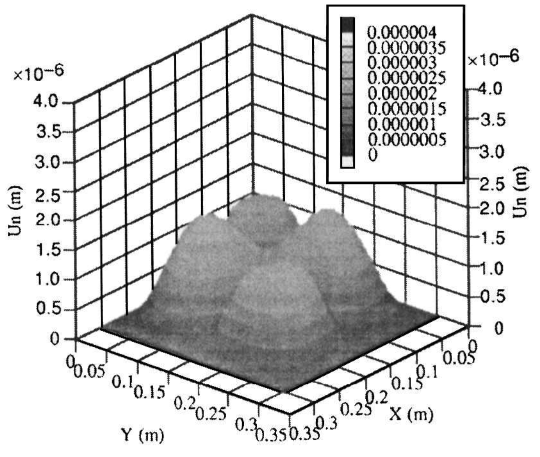


Fig. 6 Normal displacements at the top of the plate: intermediate result, $f = 270$ Hz.

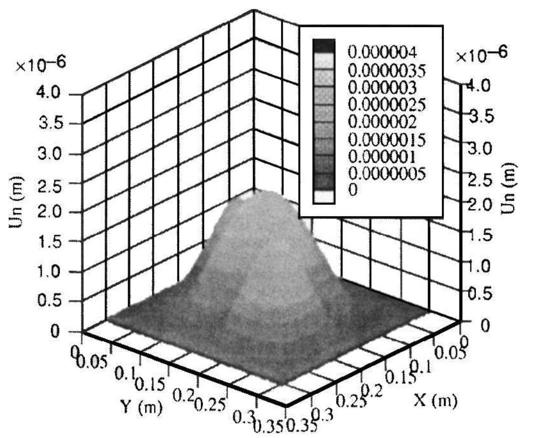


Fig. 7 Normal displacement at the top of the plate: optimal result, $f = 270$ Hz.

radiation.[32] Therefore, from the initial design stage, it is preferable to reduce the four peripheral peaks rather than the central peak for this mode of the plate for minimizing the radiated sound energy. The optimal result in Table 2 proves this fact.

In the optimal design result, the sizes of the PZT actuators are not changed very much. Another encouraging result is that the amplitudes of the voltages needed for the PZT actuators are less than 40 V, which is not high, and it still minimizes the radiated sound. By optimally designing the PZT actuators, the total radiated power at an on-resonance frequency is reduced by 20 dB or more.

To check the robustness of the optimized result, four different loading conditions previously mentioned are examined with the optimized configuration (Table 3). In the constant loading case, 20 dB reduction is achieved, whereas in the random and Bessel function

Table 3 Effect of different acoustic loading conditions on the optimal result

Loading condition	Total radiated power
A (constant)	0.9108 × E–9 (10.10 dB)
B (random)	0.9596 × E–9 (10.34 dB)
C (Bessel 0)	0.9558 × E–9 (10.32 dB)
D (Bessel 1)	0.7178 × E–9 (9.08 dB)
E (hard box)	0.6934. × E–9 (8.92 dB)

Table 4 Optimal design result for off-resonance frequency, 300 Hz

Design variables	Initial design	Intermediate result	Optimal result
$b1(x01)$	90 mm	80.09 mm	65.75 mm
$b2(y01)$	90 mm	79.01 mm	59.28 mm
$b3(x02)$	210 mm	186.46 mm	220.4 mm
$b4(y02)$	210 mm	218.39 mm	242.6 mm
$b5(r1)$	10 mm	11.04 mm	11.19 mm
$b6(r2)$	10 mm	10.42 mm	11.76 mm
$b7(t1)$	1 mm	0.88 mm	1.10 mm
$b8(t2)$	1 mm	0.92 mm	1129 mm
$b9(\lvert\phi\rvert 1)$	10 V	45.91 V	66.82 V
$b10(\lvert\phi\rvert 2)$	10 V	9.86 V	19.1 V
Objective function, W	0.903E–6 (40.08 dB)	0.2024 × E–8 (13.58 dB)	0.9253 × E–9 (10.18 dB)

Table 5 Effect of different acoustic loads on the optimal result at f = 300 Hz

Loading condition	Total radiated power
A (constant)	0.925 × 10–9 (10.18 dB)
B (random)	0.919 × 10–9 (10.16 dB)
C (Bessel 0)	0.356 × 10–8 (12.62 dB)
D (Bessel 1)	0.160 × 10–8 (12.80 dB)
E (hard box)	0.918 × 10–9 (10.15 dB)

cases, 20–21 dB reduction is observed. In the acoustically hard box case, the reduction is 21 dB.

The reason why more reduction is obtained for the acoustically hard box case rather than the random and Bessel function cases is that the excitation frequency, 270 Hz, is close to the natural frequency of the fourth mode of the plate and the acoustic pressure distribution in the acoustically hard box is described as a trigonometric function at this frequency.[33] In other words, the acoustical energy in the box is closely coupled with the structural vibration; thus this close coupling results in the reduction of noise by structural vibration control. In conclusion, with the optimal design of the piezoelectric active structure, at least 20 dB radiated noise reduction can be obtained when the noise frequency is 270 Hz.

Optimal Design at f = 300 Hz (Off Resonance)

An attempt has been made to evaluate the feasibility of actively reducing the radiated sound at an off-resonance frequency. It is known that this is a difficult problem. The excitation is at 300 Hz, the lower resonance frequency is at 272 Hz, and the next higher resonance is at 339 Hz; thus, this frequency is chosen to be between two modes. The loading condition, size of the plate, and the properties of piezoelectric material are the same as in the previous case of on-resonance excitation. Table 4 shows the initial, intermediate, and optimal results. The radii and thicknesses of piezoelectric actuators in the optimal result show a slight increase. Figures 8–11 represent the normal displacements at the top of the plate for the initial, intermediate, and optimal results, respectively. As mentioned previously regarding optimal locations of the piezoelectric actuators, when the excitation frequency is off resonance, 300 Hz, the same philosophy in reducing radiated sound energy can be applied.

To check the robustness of the optimized result, four different loading conditions are also examined with the optimized configuration (Table 5). In the constant, random, and acoustically hard box loading cases, 30 dB reduction is achieved, whereas in the Bessel function cases, 27.5–28 dB reduction is observed. It is also noticed that starting with an initial configuration with the two PZT actuators located on the diagonal of the plate, upon optimization at 270 Hz

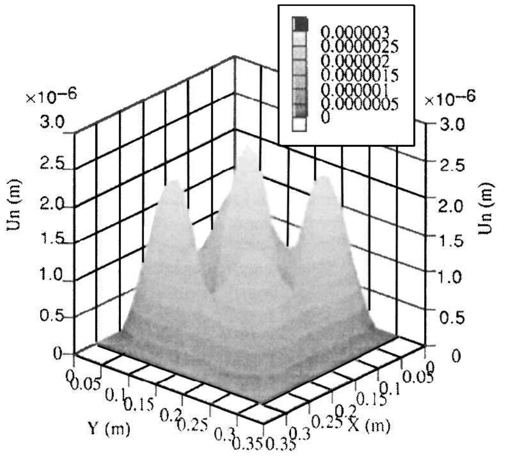


Fig. 8 Normal displacement at the top of the plate: initial design, f = 300 Hz.

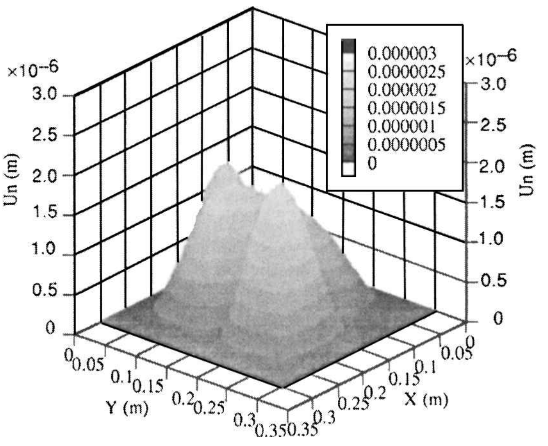


Fig. 9 Normal displacement at the top of the plate: intermediate result, f = 300 Hz.

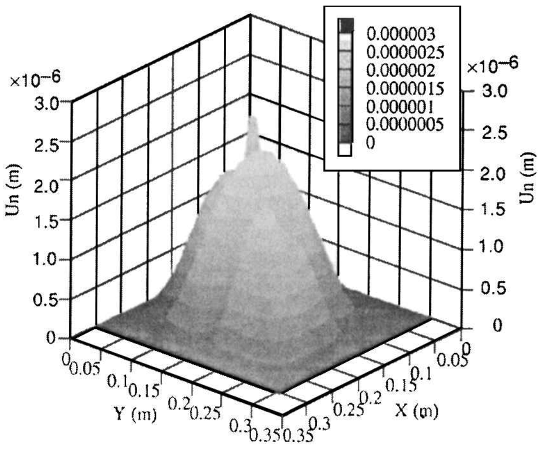


Fig. 10 Normal displacement at the top of the plate: optimal result, f = 300 Hz.

(on resonance) and at 300 Hz (off resonance), the actuators moved toward the clamped edge of the plate as may be observed from Tables 2–4. In the off-resonance case, more than 28 dB reduction in radiated sound power is achieved.

Robustness of the Design Configuration in a Bandwidth of Excitation Frequencies

According to the previous optimal design results, 20–30 dB noise reduction can be achieved at both on- and off-resonance frequencies. However, the optimally designed configuration shows different configurations of the piezoelectric actuators. This is impractical because in practice the noise frequency can change, but the location

Table 6 Total radiated sound power at different frequencies

Frequency, Hz	$\|f\|_1$, V	$\|f\|_2$, V	Object function, W
150	139.5	149.2	26.84 dB
200	164	160	14.50 dB
250	4.44	−9.09	0.90 dB
270	39.25	4.94	10.1 dB
300	80.38	9.32	15.46 dB
350	102.67	−87.26	14.64 dB
400	75.33	45.89	22.38 dB
450	8.68	−9.43	31.92 dB

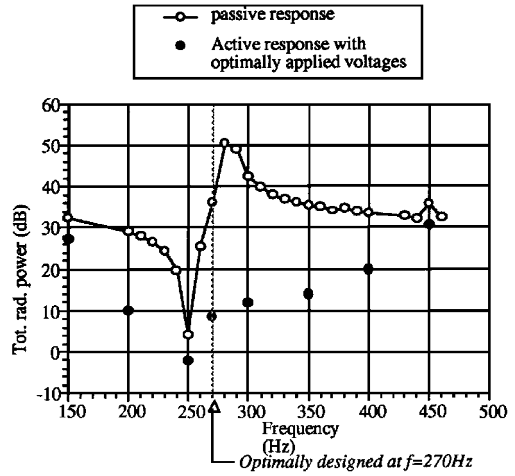


Fig. 11 Robustness of optimal design result at different frequencies.

and size of the actuators are difficult to change once they are affixed to the structure. Only the actuator voltages can be changed in practice if the noise frequency changes. Therefore, another attempt for checking the robustness of the optimal result at different frequencies has been made. At first, the optimal configuration found at $f = 270$ Hz is chosen and the noise frequency is varied from 200 to 400 Hz, and only the applied voltage on the piezoelectric actuators are optimally adjusted. The acoustic pressure loading condition is assumed to be constant. By adjusting the actuator voltages at each frequency, the frequency bandwidth in which the radiated sound can be reduced with just two actuators is determined. Table 6 shows this result. Figure 11 represents the difference between the results when the optimized actuator voltages are applied and the passive response of the structure at the different excitation frequencies. When the frequency is below 150 and or above 450 Hz, it is not possible to reduce the radiated sound by adjusting voltages on just two actuators. This defines the frequency band for the given design. With the configuration optimally designed at $f = 270$ Hz, the radiated noise can be reduced 30–10 dB in the frequency range of 200–400 Hz.

Conclusions

For reduction of radiated noise from a structure, optimal design of the sizes, locations, and applied voltages of two circular disk-shaped piezoelectric actuators on a flat plate structure has been studied. The finite element method, which uses a combination of three-dimensional piezoelectric, flat shell, and transition finite elements, is used to model the piezoelectric active structure. This approach has merits in terms of accuracy and economy in modeling even large-scale of piezoelectric active structures. The main contributions in this paper are 1) automatic mesh generation that allows for the movement of the piezoelectric actuators during optimization and automatically satisfies the appropriate electrical and mechanical boundary conditions on the actuator as it moves, 2) radiated noise reduction at on and off resonance and for a large frequency range, and 3) the use of circular disk actuators and optimization of the size and thickness of the actuators as well as their locations. The automatic mesh and boundary condition generation program is based on the idea of moving mesh templates for complicated regions near piezoelectric actuators.

The objective function that is minimized in the optimization procedure is the total sound power radiated from the structure. Significant reduction of radiated noise is observed both at on- and off-resonance frequencies. Optimal locations of the piezoelectric actuators tend to be near the corners of the plate rather than the center of the plate. This makes sense because edges or corners of flat plates contribute significantly to radiation at a fixed frequency. To find the applicability of this approach to a band of frequencies rather than just at a single frequency, the excitation frequency is varied and it is observed that good reduction of the radiated noise can be achieved by adjusting just the voltages applied to the actuators. In the optimal configurations of the piezoelectric actuators, the required voltages are low, and hence it is eminently practical.

To resolve the difficulty of finding the true acoustic pressure loading in structural acoustic problems, the load distribution is assumed to be a constant and several different patterns of pressure loading conditions are examined to verify the validity of the result. When different acoustic loading functions are tested, some degradation of the objective function is observed but the results are still very acceptable.

In the optimization procedure, a nonderivative based unconstrained minimization technique is used because there is no overhead to evaluate derivatives of the objective function with respect to the design variables. However, in contrast, this technique takes too much time to converge. Therefore, sensitivity analysis for the radiated sound power and the finite element equation of piezoelectric active structures with variable locations of the actuators is necessary.

Appendix A: Automatic Mesh Generation Algorithm

Node Generation

1) Generate the nodes for templates 1 and 2 from the given radii and locations of piezoelectric devices and store them. Nodes are automatically generated when the locations are given. The size of the template is proportional to the radius of the piezoelectric device and is of rectangular shape.

2) Project the outlines of the two templates onto the four sides of the plate. The attribute arrays for the discretized points are according to the following logic: 0 = ordinary point (no intersection with template), 1 = template 1 territory (this point interferes with template 1), 2 = template 2 territory (this point interferes with template 2), and 3 = territory between template 1 and 2 (when two templates are overlapping in the other direction).

3) Divide each side and assign attributes for discretized points. When the point attribute is 0, the distance of points that have 0 attribute consecutively is divided by the given minimum element size and the number of division is truncated. When the attribute is 1 or 2, coordinates are copied from the corresponding template and when the attribute value is 3, the division is performed in the same manner as the 0 attribute case.

4) If two templates contact one another in the x direction, make X3PT that takes into account the discretized points at the contact line.

5) Sweep the IX1CH values as well as IY1CH to generate nodes for the plate elements.

Element Generation

Element generation is somewhat simpler than node generation. Two element templates are generated from the leading node array of each template. The leading node array holds the nodal numbers of the left side of the template and is generated during the node generation procedure. While sweeping indices in x and y directions, if there is a template, copy elements from the element template corresponding to the attribute and otherwise generate plate elements.

Appendix B: Material Properties

1) Aluminum: (structure material) Young's modulus = 68 GPa, Poisson's ratio = 0.32, density = 2800 Kg/m^3.

2) PZT-5H: (piezoelectric devices) density = 7500 kg/m^3. The elastic property matrix with constant electric field is

$$C = \begin{bmatrix} 12.6 & 7.95 & 8.41 & 0 & 0 & 0 \\ & 12.6 & 8.41 & 0 & 0 & 0 \\ & & 11.7 & 0 & 0 & 0 \\ & & & 2.33 & 0 & 0 \\ & \text{sym} & & & 2.3 & 0 \\ & & & & & 2.3 \end{bmatrix} \times 10^{10}$$

The piezoelectric strain matrix is

$$h^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 17 \\ 0 & 0 & 0 & 0 & 17 & 0 \\ -6.5 & -6.5 & 23.3 & 0 & 0 & 0 \end{bmatrix}$$

The dielectric property matrix with constant strain is

$$b = \begin{bmatrix} 1.503 & 0 & 0 \\ 0 & 1.503 & 0 \\ 0 & 0 & 1.3 \end{bmatrix} \times 10^{-8}$$

References

[1] Cremer, L., Heckl, M., and Ungar, E., *Structure-Borne Sound*, 2nd ed., Springer–Verlag, New York, 1988.

[2] Belegundu, A. D., Salagame, R. R., and Koopmannn, G. H., "A General Optimization Strategy for Sound Power Minimization," *Structural Optimization*, Vol. 8, Oct. 1994, pp. 113–119.

[3] Lamancusa, J. S., "Numerical Optimization Techniques for Structural-Acoustic Design of Rectangular Panels," *Computers and Structures*, Vol. 48, Aug. 1993, pp. 661–675.

[4] Crawley, E., and de Luis, J., "Use of Piezoelectric Actuators as Elements of Intelligent Structures," *AIAA Journal*, Vol. 25, No. 10, 1987, pp. 1373–1385.

[5] Lee, C. K., "Theory of Laminated Piezoelectric Plates for the Design of Distributed Sensors/Actuators: Part I: Governing Equations and Reciprocal Relationships," *Journal of the Acoustical Society of America*, Vol. 87, March 1990, pp. 1144–1158.

[6] Varadan, V. V., Bao, X., and Varadan, V. K., "Smart Plates for Noise Reduction Using MEMS Technology," Smart Structures and Materials 1996: Smart Electronics and MEMS, edited by V. K. Varadan and P. J. McWhoorter, *Proceedings of the SPIE*, Vol. 2722, 1996, pp. 230–240.

[7] Clark, R. L., and Fuller, C. R., "Experiments on Active Control of Structurally Radiated Sound Using Multiple Piezoceramic Actuators," *Journal of the Acoustical Society of America*, Vol. 91, No. 6, 1992, pp. 3313–3320.

[8] Tzou, H. S., and Tseng, C. I., "Distributed Piezoelectric Sensor/Actuator Design for Dynamic Measurement/Control of Distributed Parameter Systems: A Piezoelectric Finite Element Approach," *Journal of Sound and Vibration*, Vol. 138, April 1990, pp. 17–34.

[9] Lindberg, R. E., "Actuator-Placement Considerations for the Control of Large Space Structures," NRL 8675, May 1983.

[10] Lim, K. B., "Method for Optimal Actuator and Sensor Placement for Large Flexible Structures," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 1, 1992, pp. 49–57.

[11] Igusa, T., Xu, K., and Warnitchai, P., "Optimal Locations and Gains of Sensors and Actuators for Feedback Control," *Collection of Technical Papers: AIAA/ASME Structures, Structural Dynamics, and Materials Conference*, Pt. 6, AIAA, Washington, DC, 1993, pp. 3137–3145 (AIAA Paper 93-1660).

[12] Hamidi, M., and Juang, J.-N., "Optimal Control and Controller Location for Distributed Parameter Elastic Systems," *Proceedings of the 20th IEEE Conference on Decision and Control*, 1981, pp. 502–506.

[13] Clark, R. L., and Fuller, C. R., "Optimal Placement of Piezoelectric Actuators and Polyvinylidene Fluoride Error Sensors in Active Structural Acoustic Control Approaches," *Journal of the Acoustical Society of America*, Vol. 92, No. 3, 1992, pp. 1521–1533.

[14] Wang, B.-T., Burdisso, R. A., and Fuller, C. R., "Optimal Placement of Piezoelectric Actuators for Active Structural Acoustic Control," *Journal of Intelligent Materials and Structures*, Vol. 5, No. 1, 1994, pp. 67–77.

[15] Kim, J., Varadan, V. V., and Varadan, V. K., "Finite Element-Optimization Methods for the Active Control of Radiated Sound from a Plate Structure," *Journal of Smart Materials and Structures*, Vol. 4, No. 4, 1995, pp. 318–326.

[16] Varadan, V. V., Kim, J., and Varadan, V. K., "Optimal Design of Enclosures for Active Noise Control and Isolation," Smart Structures and Materials 1996: Mathematics and Control in Smart Structures, edited by V. V. Varadan, *Proceedings of the SPIE*, Vol. 2442, 1995, pp. 470–475.

[17] Dimitriadis, E. K., Fuller, C. R., and Rogers, C. A., "Piezoelectric Actuators for Distributed Vibration Excitation of Thin Plates," *Journal of Vibration and Acoustics*, Vol. 113, No. 1, 1991, pp. 100–107.

[18] Tzou, H. S., and Zhong, J. P., "Theory on Hexagonal Symmetrical Piezoelectric Thick Shells Applied to Smart Structures," Design Engineering Div., Vol. DE-34, American Society of Mechanical Engineers, 1991, pp. 7–15.

[19] Reddy, J. N., "A Generalization of Two-Dimensional Theories of Laminated Composite Plates," *Communications in Applied Numerical Methods*, Vol. 3, No. 3, 1987, pp. 173–180.

[20] Reddy, J. N., and Robins, D. H., Jr., "Structural Theories and Computational Models for Composite Laminates," *Applied Mechanics Reviews*, Vol. 47, Pt. 1, 1994, pp. 147–170.

[21] Kim, J., Varadan, V. V., and Varadan, V. K., "Finite Element Modeling of Structures Including Piezoelectric Active Devices," *International Journal of Numerical Methods in Engineering* (to be published).

[22] Chin, L. C., Varadan, V. V., and Varadan, V. K., "Finite Element Methods for Numerical Simulation of the Actuator Performance of a Composite Transducer Array," *Proceedings of the ADPA/AIAA/ASME/SPIE Conference on Active Materials and Adaptive Structures*, edited by G. J. Knowles, Inst. of Physics, Philadelphia, 1991, pp. 633–637.

[23] Kim, J., Varadan, V. V., and Varadan, V. K., "Finite Element Modeling of a Finite Piezoelectric Sensor/Actuator Embedded in a Fluid-Loaded Plate," Smart Structures and Materials 1994: Mathematics and Control in Smart Structures, *Proceedings of the SPIE*, Vol. 2192, 1994, pp. 271–275.

[24] Zienkiewicz, O. C., and Taylor, R. L., *The Finite Element Method*, Vol. 2, 4th ed., McGraw–Hill, New York, 1991.

[25] Surana, K. S., "Transition Finite Elements for Three-Dimensional Stress Analysis," *International Journal Numerical Methods in Engineering*, Vol. 15, 1980, pp. 991–1025.

[26] Bathe, K. J., *Finite Element Procedures in Engineering Analysis*, Prentice–Hall, Englewood Cliffs, NJ, 1982.

[27] Press, W. H., Flannery, B. P., Teukolsky, S. A., and Vetterling, W. T., *Numerical Recipes in C*, Cambridge Univ. Press, New York, 1988.

[28] Phelan, D. G., and Haber, R. B., "Sensitivity Analysis of Linear Elastic Systems Using Domain Parameterization and a Mixed Mutual Energy Principle," *Computer Methods in Applied Mechanical Engineering*, Vol. 77, No. 1–2, 1989, pp. 31–59.

[29] Suh, C. H., and Radcliffe, C. W., *Kinematics and Mechanisms Design*, Wiley, New York, 1978.

[30] Kim, J., Varadan, V. V., and Varadan, V. K., "Finite Element Modeling of Smart Cantilever Plate and Comparison with Experiments," *Journal of Smart Materials and Structures*, Vol. 5, No. 2, 1996, pp. 165–170.

[31] Leissa, A., *Vibration of Plates*, Acoustical Society of America, 1993.

[32] Fahy, F., *Sound and Structural Vibration*, Academic, New York, 1985.

[33] Kinsley, L. E., Frey, A. R., Coppens, A. B., and Sanders, J. V., *Fundamentals of Acoustics*, 3rd ed., Wiley, New York, 1982.

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